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# Review of station keeping strategies for elliptically orbiting constellations in along-track formation

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**Abstract**

Three techniques for station keeping an orbiting constellation of satellites in an elliptical orbit are developed: (1) based on an application of the linearized Tschauner–Hempel (TH) equations for the motion of a daughter satellite relative to a reference (mother) satellite together with the linear quadratic regulator (LQR) control strategy which can be used in a piecewise adaptive manner; (2) since the mathematical model is inherently nonlinear and time varying, a control law based on a non-linear Lyapunov function is applied to daughter satellites' osculating orbital elements; (3) by carefully selecting relative orbital design parameters so that the relative secular drifts due to the non-spherical Earth's perturbation in the longitude of the ascending node, the argument of perigee and mean anomaly could vanish or be constrained to a desired value.


*Keywords:* Station keeping; Elliptically orbiting constellations; Along-track formations

## 1. Introduction

Of several designs proposed for the NASA Auroral Cluster Observation System mission is an along-track formation in an elliptical orbit of up to four spacecraft with constant distances between adjacent satellites.

A novel idea proposed in Tan et al. (1999) would involve an impulsive maneuver at perigee that would cause a small shift in the direction of the semi-major axis of the daughter satellite with respect to the

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original orbit (of the mother satellite). Without perturbations and subsequent control effort this separation distance could be maintained with the drift from the nominal value remaining of the order of ±2%. In the presence of perturbations (mainly from the Earth's oblateness) additional station keeping control would be required to counteract the resulting secular drift in the average separation distance.

## 2. Application of the linearized Tschauner–Hempel equations with an LQR control strategy

The Tschauner–Hemple equations (Tschauner, 1967) describe the motion of a daughter spacecraft close to rendevous with a reference or mother spacecraft in a nominal elliptical orbit. These equations can be linearized and recast in the familiar state-space representation as:

$$x' = Ax + Bu \quad \text{where } x = (\xi\eta\zeta\xi'\eta'\zeta')^{\mathrm{T}}, \quad ' = \mathrm{d}()/\mathrm{d}f \tag{1}$$

and $\xi$, $\eta$ and $\zeta$ are non-dimensionalized coordinates centered at the target or reference spacecraft. $\xi$ describes cross track (outward radial) motion, $\eta$-along track, and $\zeta$ out of the nominal orbit plane of the target spacecraft. The non-zero elements of the state matrix, $A$ are: $a_{14} = a_{25} = a_{36} = 1 = -a_{63}$; $a_{45} = 2 = -a_{54}$; and $a_{31} = 3\mu r/h^2$. $r$ refers to the orbital radius, $h$ the angular momentum per unit mass, $\mu$ the gravitational constant (universal gravitational constant times the Earth's mass), and $f$ is the true anomaly. The variable term in the state matrix can be adjusted in a number of ways:

1. When it is assumed that $r$ remains constant (i.e. equal to $r(\theta) = h^2/\mu$), true for a circle and relatively short displacements, then the term becomes equal to 3.
2. If the simulation is started at perigee or apogee then evaluate $r$ at perigee or apogee, respectively, and treat as constant for sufficiently short time thereafter.

The last two matrices in Eq. (1) can be identified as the control influence matrix $B$, and the control input matrix $u = [a_\xi, a_\eta, a_\zeta]^{\mathrm{T}}$, in terms of the control accelerations. If the control accelerations are not provided directly along the $\xi$,$\eta$, $\zeta$ axes then the $B$ matrix (e.g. $C_i$ values, or more complicated arrangements) can be adjusted accordingly. For simplicity here, assume $u_1 = c_1\ a_\xi$, $u_2 = c_2\ a_\eta$, $u_3 = c_3\ a_\zeta$.

For the application of the linear quadratic regulator, assume that the state matrix is, at least, piecewise constant, and that the performance index (Athans and Falb, 1966)

$$J = \int_0^\infty [x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru]\,\mathrm{d}t \tag{2}$$

Assuming all states are immediately available and in the absence of noise, a parametric study was undertaken by using various weighting functions. A typical result is shown in Fig. 1 of Bainum et al. (2002), shown here as Fig. 1.

The results presented assume that the initial LQR correction begins near perigee at a true anomaly angle of 45°. The periodic system $A$ matrix is evaluated at that true anomaly angle. If the responses occur in a relatively short time interval it is assumed that this value could be used throughout the maneuver (the case considered here). The state here is defined as $(x, y, z, \dot{x}, \dot{y}, \dot{z})$. The center of this moving coordinate system is taken at the nominal position of any daughter satellite in the orbit. Thus a displacement of 1 km along track actually represents a displacement of $d_{\text{mother–daughter}} + 1$ km along track, where $d_{\text{mother–daughter}}$ is the desired mother–daughter separation distance for this application. Here an out-of-plane error in position of 1 km is given at the 45° true anomaly point, in addition to a 1 km error along track. Fig. 1 depicts consistence as it relates to input error. Furthermore, damped simple harmonic motion (SHM) is demonstrated as predicted by the out-of-plane Tschauner–Hempel equation.

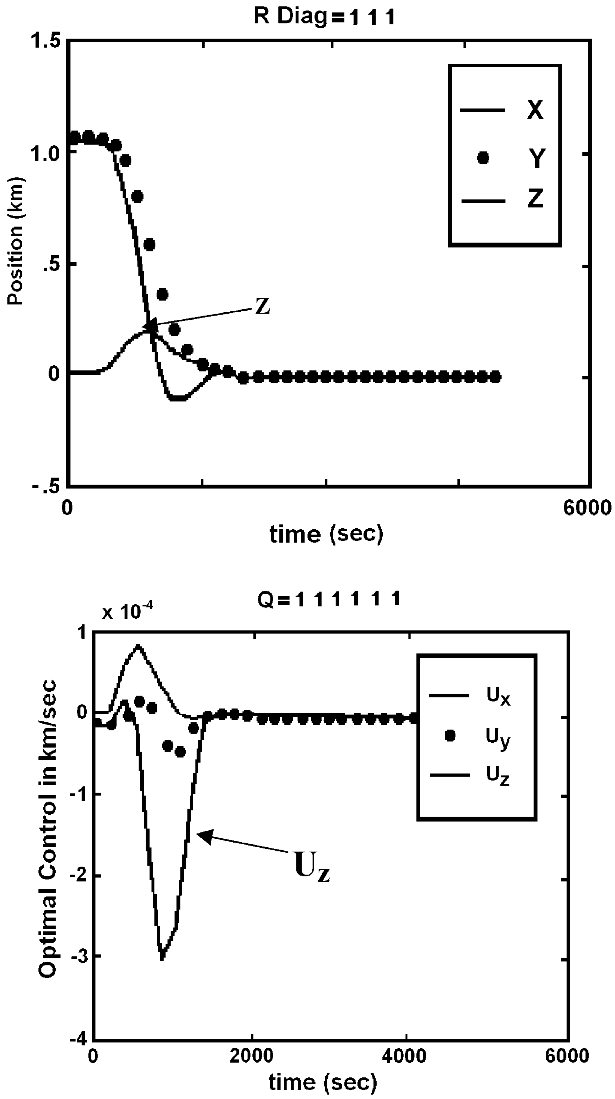


Fig. 1. LQR transient responses—in-plane and out-of-plane initial displacements.

## 3. Nonlinear control law based on Lyapunov function applied to the osculating orbital elements

This is a nonlinear control law based on the non-linear mathematical model. Comparing to the control strategy described above, there is no error introduced from linearization. The variational equations of Gauss provide a convenient set of equations relating the effect of a control acceleration vector **u** to the osculating orbital element time derivatives (Battin, 1987):

$$\dot{a} = (2a^2/h)(e\sin f u_r + (p/r)u_\theta) \quad (3)$$

$$\dot{e} = [p\sin f u_r + ((p+r)\cos f + re)u_\theta]/h \quad (4)$$

$$\dot{i} = [(r\cos\theta)/h]u_h \tag{5}$$

$$\dot{\Omega} = [(r\sin\theta)/(h\sin i)]u_h \tag{6}$$

$$\dot{\omega} = \frac{1}{he}[-p\cos f u_r + (p+r)\sin f u_\theta] - \frac{r\sin\theta\cos i}{h\sin i}u_h \tag{7}$$

$$\dot{M} = n + [\eta/(he)][(p\cos f - 2re)u_r - (p+r)\sin f u_\theta] \tag{8}$$

We define $\mathbf{x} = (a \quad e \quad i \quad \Omega \quad \omega \quad M)'$ as the state variable vector and $\mathbf{u} = (u_r \quad \mu_\theta \quad u_h)'$ as the control acceleration vector, written in the Local-Vertical-Local-Horizontal frame, $f$ is the true anomaly, $r$ is the scalar orbit radius, $p = b^2/a$ is the semi-latus rectum, $\theta = \omega + f$, and $h = \sqrt{p\gamma}$, $\eta = \sqrt{1-e^2}$, $n = \sqrt{\gamma/a^3}$. Incorporating the $J_2$ influence, Eqs. (3)–(8) can be written as

$$\dot{x} = B(x)u + D(x) \quad \text{where } D(x) = D(J_2, R_e, n, a, e, i) \tag{9}$$

It is found out that the elements of $D$ are either zeros or very small as compared with non-zero elements of $B(x)u$, therefore, $D(x)$ is treated as a minor disturbance. If the osculating orbital elements of the mother satellite are $x_1$, the required osculating orbital elements of the first daughter satellite are $x_2$, then

$$\Delta x = x_2 - x_1 \quad \text{i.e.} \quad x_2 = x_1 + \Delta x \tag{10}$$

Assuming that the actual osculating orbital elements of the first daughter satellite are $x_{2d}$, then

$$\delta x = x_{2d} - x_2 \quad \text{i.e.} \quad x_{2d} = x_2 + \delta x \tag{11}$$

We define a Lyapunov function as

$$V = \frac{1}{2}(a + be^{-\alpha t}) \bullet \delta x^{\mathrm{T}}\delta x \quad \text{where } a > 0, b > 0, \alpha > 0, \quad \text{therefore, } V > 0 \tag{12}$$

then

$$\dot{V} = -\frac{1}{2}b\alpha e^{-\alpha t}\delta x^{\mathrm{T}}\delta x + (a + be^{-\alpha t})\delta x^{\mathrm{T}}\delta\dot{x} \tag{13}$$

$$\delta x = x_{2d} - x_2 = x_{2d} - x_1 - \Delta x \quad \text{see Eqs. (10) and (11)}$$

where

$$\therefore \quad \delta\dot{x} = \dot{x}_{2d} - \dot{x}_1 \quad \text{note that } \Delta x \text{ does not vary with time}$$

After substitution, it can be shown that $\dot{V}$ is negative semi-definite (Bainum et al., 2002).

A typical result with $J_2$ included and for initial errors in the daughters "$a$" of 1 km, and small errors in "$e$" and "$i$" shows that the osculating orbital elements converge smoothly and that the maximum control efforts are less than $10^{-4}$ m/s$^2$ (Fig. 4 of Bainum et al. (2002)), shown here as Fig. 2.

## 4. Selecting orbital design parameters so that relative secular drifts remain small

The analytic solutions of the perturbation equations using the mean orbital elements for the motion of a spacecraft are given in (Duan and Bainum (2003)) in terms of long and short periodic oscillations as well as secular effects.

The possibility that the relative secular drifts due to the non-spherical Earth's perturbation in the longitude of the ascending node, the argument of perigee and mean anomaly could vanish or be constrained to a desired value is discussed. A general set of solutions is introduced for this problem. Thus, it is possible to

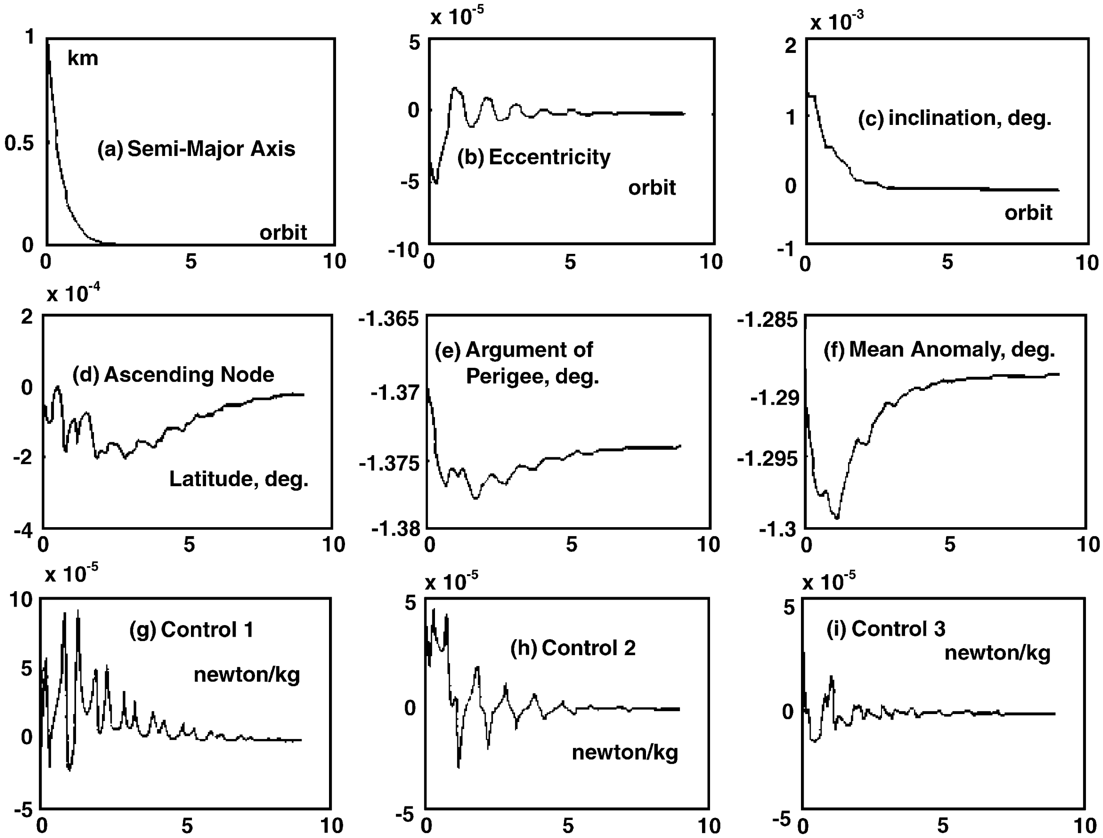


Fig. 2. Difference of orbital elements between adjacent satellites and control efforts.

choose design parameters to satisfy different requirements for formation flying and constellation station keeping.

Let the difference between the secular drift rates of two neighboring orbits (long term drift rates of the longitude of the ascending node, the argument of perigee and the mean anomaly) be: $\Delta\Omega$, $\Delta\omega$ and $\Delta M$. We can obtain the following:

$$\Delta\Omega = \frac{\partial(\Omega_1 + \Omega_2 + \cdots)}{\partial a}\Delta a + \frac{\partial(\Omega_1 + \Omega_2 + \cdots)}{\partial e}\Delta e + \frac{\partial(\Omega_1 + \Omega_2 + \cdots)}{\partial i}\Delta i + H.O.T \tag{14}$$

$$\Delta\omega = \frac{\partial(\omega_1 + \omega_2 + \cdots)}{\partial a}\Delta a + \frac{\partial(\omega_1 + \omega_2 + \cdots)}{\partial e}\Delta e + \frac{\partial(\omega_1 + \omega_2 + \cdots)}{\partial i}\Delta i + H.O.T \tag{15}$$

$$\Delta M = \frac{\partial(n + M_1 + M_2 + \cdots)}{\partial a}\Delta a + \frac{\partial(n + M_1 + M_2 + \cdots)}{\partial e}\Delta e + \frac{\partial(n + M_1 + M_2 + \cdots)}{\partial i}\Delta i + H.O.T \tag{16}$$

We can rewrite the first order form of above equation as $A\vec{x} = \vec{b}$, where

$$\begin{aligned} \vec{x} &= [\,\Delta a \quad \Delta e \quad \Delta i\,]^{\mathrm{T}} \\ \vec{b} &= [\,\Delta\Omega \quad \Delta\omega \quad \Delta M\,]^{\mathrm{T}} \end{aligned} \tag{17}$$

For all the relative secular drifts to vanish ($\vec{b}=0$), we look for non-zero solutions of $A\vec{x}=\vec{b}$. We can consider the following situations:

Case 1: $\Delta\Omega = 0$ and $\Delta(\omega + M) = 0$;
Case 2: $\Delta\Omega = \Delta\omega = \Delta M = 0$;
Case 3: $\Delta M = 0$ and $\Delta(\omega + \Omega) = 0$ etc.

Analytic solutions for different cases can be formulated based on the general solution. For example, Eqs. (18) and (19) show a set of simplified solutions for case 1.

$$\Delta a \cong (1 + 5\cos^2 i)\left[-\frac{4}{3} - (1-e^2)^{1/2}\right]\frac{A_2 e}{a(1-e^2)^3}\Delta e \tag{18}$$

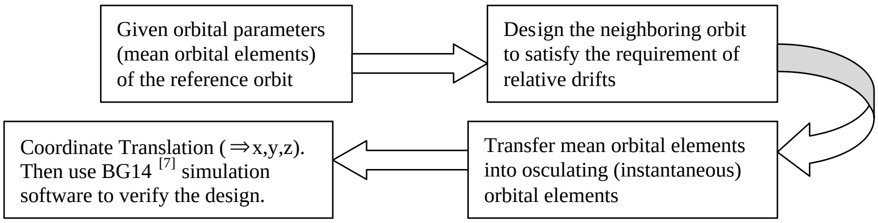


Fig. 3. Simulation procedure.

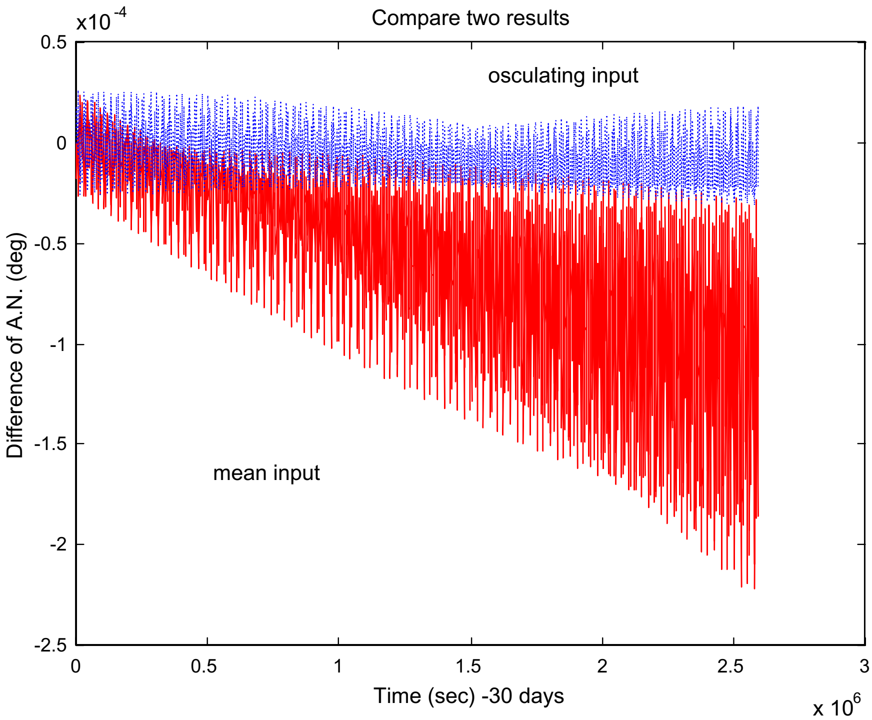


Fig. 4. The relative drift of the longitude of the ascending node.

$$\Delta a = \left\{ \tan i \cdot \left[ \left(2 - \frac{5}{2}\sin^2 i\right) + \frac{3}{4}\left(1 - e^2\right)^{1/2}\left(1 - \frac{3}{2}\sin^2 i\right) \right] + \sin i \cos i[-3(1-e^2)^{1/2} - 5] \right\} \frac{\frac{2}{3}A_2}{a(1-e^2)^2}\Delta i \tag{19}$$

The BG14 Advanced Simulation Development System (ASDS), a high precision numerical simulation model, is used to verify the correctness of the algorithms and the design. Developed by McDonnell Douglas Aerospace, BG14 ASDS provides highly accurate propagation of orbits over both short and long time intervals and is suited for simulating many problems. This simulation software is independent of the algorithms and the design proposed in this research, and provides an independent verification of the accuracy of the computed results.

Fig. 3 demonstrates the simulation procedure. As an example here, Case 1 is considered. The simulation tries to verify the effectiveness of causing the relative drift of the longitude of the ascending node and also the relative drift of the sum of the argument of perigee and mean anomaly to vanish, and, finally, the effectiveness of maintaining the relative distance between two satellites on two neighboring orbits.

A typical numerical result is shown in Figs. 4–6 where the relative drifts and the desired 1 km separation between a daughter and mother satellite can be very well maintained during a 30 day response based on the use of the osculating orbital elements as input to the orbital propagator software. However, the design using the mean elements directly is divergent, contrasting with the results based on the osculating elements as input. It can be observed clearly that although the mean input and the osculating input come from the same design for the relative orbits, a small variation in nominal orbital parameters can result in a noticeable difference in the maintenance of the formation or constellation.

In summary, by carefully selecting relative orbital design parameters, the relative drifts in the longitude of the ascending node, the argument of perigee and mean anomaly, due to the $J_2$, $J_4$ effects or the

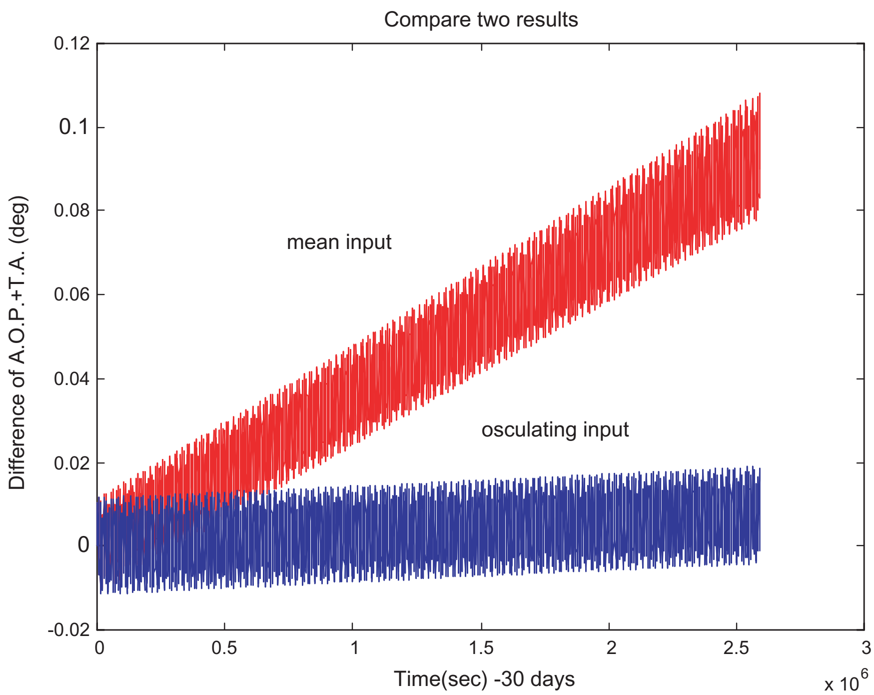


Fig. 5. The relative drift of the sum of the argument of perigee and the true anomaly.

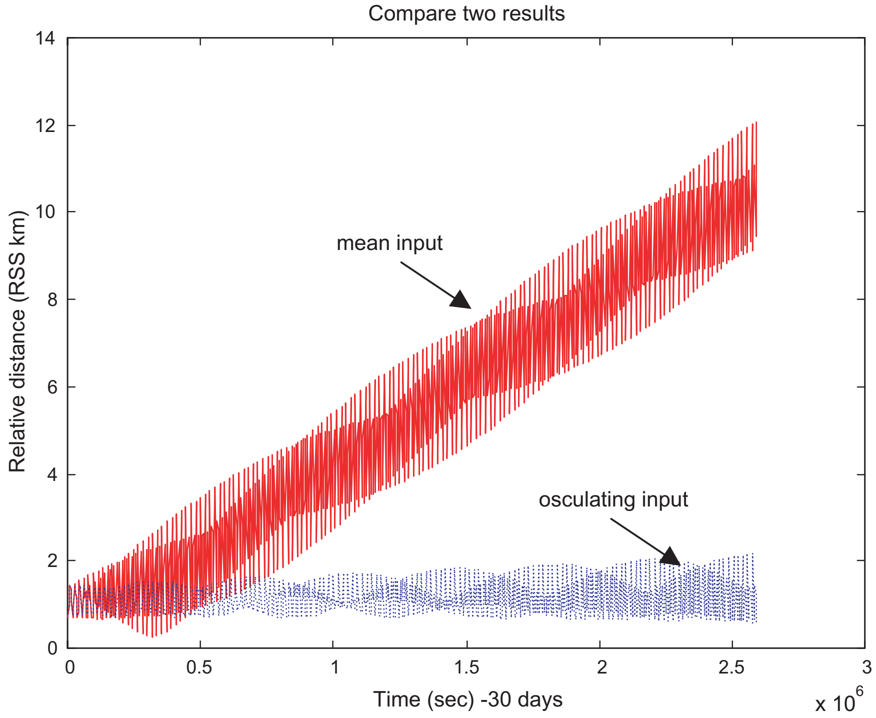


Fig. 6. Time history of separation distance.

combination of them could vanish or be controlled to a desired value. Therefore, a general design could be provided for formation flying and constellation station keeping. There could be a significant savings in energy consumption for the whole life cycle of the mission since this kind of relative orbits may need less energy consumption.

## 5. Conclusions

Simulation results for the first two station keeping techniques show that the responses to initial errors converge smoothly with the control energy at a low level. The LQR-TH approach incorporates the robustness advantages of an LQR technique, while the Lyapunov approach is shown to result in an asymptotically stable nonlinear system. High precision simulations show the effectiveness of the third approach in choosing orbital design parameters to minimize the effects of secular drifts due to perturbations.

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